Monetization Strategy Innovation in Mobile App Market

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Version: March 19, 2021

This paper explores a type of non-technological innovation, monetization strategy innovations in the mobile app market, and examines their adoption and impacts to better understand what types of apps choose which monetization strategies. This paper mainly exploits adoption of a novel monetization strategy, in-app currency. This paper finds suggestive evidence of positive peer effects within app category in adoption of the novel strategy and the association of the adoption decisions and profits with app ratings. Moreover, better performance by later entrant adopters relative to the early ones is to some degree driven by better app ratings of the late entrants. Across app categories, adoption of the novel strategy has been more common in some categories than others, and adopters do better than non-adopters in categories where the strategy is more popular. Throughout the paper, the results suggest that monetization innovations may not benefit all types of apps, which could be taken into consideration in designing policy programs to promote innovations.

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1 Introduction

Recent literature has examined non-technological innovations, such as organizational structure, marketing or other corporate strategies, and management practices (e.g. Bloom and Van Reenen, 2010; Battisti, Gallego, Rubalcaba, and Windrum, 2014). This paper explores a type of non-technological innovation, namely innovations in ways to monetize a user base in the mobile app market. Using the app-level data, I examine the adoption of monetization innovations and the impacts of innovation adoption to better understand what types of apps choose which monetization strategies.

Adoption of innovations could spread in the market. I find some positive peer effects that when more apps adopt innovation within some reference group, a later entrant is also more likely to adopt the innovation. However, I cannot identify the underlying mechanism since different mechanisms can generate an observationally equivalent aggregate-level diffusion pattern.[1] This peer effect does not narrow down to a single implication, but suggests a few non-mutually-exclusive possibilities, including information imperfection, learning across cohorts, and growing consumer interests in apps using a novel monetization strategy.

Motivated by upward trends in adoption of both a novel strategy, "in-app currency," and app ratings, app quality may be another factor that affects innovation adoption. Differences in app quality across apps could partition the population between those for whom adoption is optimal and those for whom it is not. I find that an app with a higher rating is more likely to adopt in-app currency, and find the association of app ratings with adoption of other monetization strategies.

Performance gains from innovations have been examined in a line of literature (e.g. Bloom and Van Reenen, 2007; and Geroski, Machin, and Van Reenen, 1993). The relationship between innovation adoption and performance is a relevant question to understand how apps as a profit-maximizers choose monetization strategies and what types of apps adopt a novel strategy. First, I find better performance by in-app currency adopters than non-adopters. Then, I ask whether early market entrants gain more from adoption than later entrants and what types of apps benefit more from adoption. I find that better performance by the later entrant adopters relative to the early ones is to some degree driven by app ratings though I cannot rule out other drivers that may be correlated with both app rating and performance, such as consumers' growing interests in the novel monetization strategy. I also find that adoption of a novel strategy has been more common in some categories than others and that adopters do better than non-adopters in categories where the novel strategy is more popular.

There are a few benefits of focusing on monetization strategy innovations in the mobile app market. First, novel monetization strategies have been introduced in the early stage of this relatively new industry. "In-app purchase (IAP)," a class of strategies by which users are charged for additional features and contents within an app, is newer relative to "in-app advertisement (IAA)," a strategy of showing ads within an app. Within finer classification of the IAP strategy, novel relative to the other subcategories is "in-app currency," a strategy where users can buy in-app money that can be used within an app like a currency. My

[1] The diffusion of innovation adoption has generally been known to follow a logistic distribution (Griliches, 1957; Geroski, 2000; Hall, 2006).

empirical analyses exploit these monetization strategy innovations. Second, the app market provides variations across entrant cohorts and across app categories for many comparable adopters and non-adopters of the innovations. Third, I can abstract from a dynamic (delayed) adoption process given costly and rare switching between monetization strategies and the short-living nature of mobile apps.

In order to gain more economic insights, I develop a simple model of app quality heterogeneity, where apps make rational adoption choices between novel and conventional monetization strategies upon entering the market. When making the adoption decision, an app faces the trade-off between higher fixed costs and better performance of the novel strategy, which leads to a threshold quality below which the app adopts the old strategy. The model suggests that the novel monetization strategy selectively benefit and are adopted by apps of high quality, supported by estimation of a logistic model of adoption choice between novel and old monetization strategies, i.e. IAP and IAA respectively. The estimation also suggests adoption of both strategies particularly by high-quality apps.

This paper's main contribution is examination of an unexplored non-technological innovation, i.e. monetization strategy innovations, and a better understanding of the adoption and impacts of these particular innovations.

The remainder of this paper is structured as follows. Section 2 describes data. Section 3 presents descriptive statistics and empirical analyses. Section 4 presents a model of app's monetization strategy adoption choices. Section 5 presents empirical analyses based on the model. Section 6 concludes.

2 Data

Two main data sources are MixRanks and Mobile Innovation Group.[2] Data on ratings, monetization strategies, and top chart rankings are from Mobile Innovation Group. The sample is 6,158 free apps with IAP that were released from February 2010 until September 2014 and were ranked on the top chart at least once. The database provides app release date (i.e. app's market entry timing that defines entrant cohorts) and 23 app categories.[3] The database provides whether an app offers IAP and the seven subcategories of the IAP strategy: freemium to add features, freemium to remove features, subscription to add features, subscription to remove features, in-app credit, in-app currency, and bundled in-app purchases. A novel IAP strategy among the IAP subcategories is "in-app currency" where users can buy in-app money, used as a currency on access to features or contents (e.g. character upgrades, game items, etc.).[4] This paper defines "freemium" as a strategy by which users can use mobile apps free of charge to some extent but are charged to add (or remove) some

[2]Information about the Mobile Innovation Group is available at http://mig.stanford.edu. Information about MixRank is available at https://mixrank.com

[3]App categories are books, business, catalogs, education, entertainment, finance, food & drink, games, health & fitness, lifestyle, medical, music, navigation, news, photo & video, productivity, reference, shopping, social & networking, sports, travel, utilities, and weather.

[4]An example of in-app currency would be game coins used for extra life or items within a game, and different amounts of coin purchase options in the example app are shown in the third screenshot of Figure A.1.

features.[5] "Subscription" refers to a strategy by which users are charged a recurring price to add (or remove) features.[6] "In-app credit" is a strategy where users need to buy credits that are deducted for each use of features.[7] Lastly, "bundled in-app purchases" refers to a strategy by which features that are sold separately are also bundled and sold as a package.[8] These subcategories are non-mutually-exclusive in the sense that an app can adopt more than one strategy, and examples would be the use of both freemium to remove features and in-app currency in Figure A.1 and the use of freemium to add features and bundled IAP in Figure A.2.

As a proxy for app quality, the average of ratings has a mean of 4.1 (out of 5 stars) and a standard deviation of 0.61. The database provides IAP revenue range estimates. Revenues from IAP have been known to be predictable to some extent from the top chart rankings. As a performance measure, I use an indicator of whether an app has generated revenues greater than $5,000 until June 2015 (so at least 6 months are given to the latest entrants in the sample). 16% of the sample apps generated more than $5,000. The database also provides the top chart app rankings based on downloads and revenues, the inverse of which are later used as measures of incoming users and profits to reflect the highly skewed earnings (i.e. concentrated on the upper tail) across rankings. As a measure for congestion, I use the number of non-monetizing mobile apps (e.g., corporate apps, banking apps, shopping apps, etc.). This is used mainly for the empirical analysis in Section 3.

I match 10,772 apps (from February, 2010 to June, 2013) of the Mobile Innovation Group app data to the MixRank's software developments kits (SDK) data that provide the list of SDKs used in an app, including a SDK for IAA (in-app ads). Constrained by the MixRank data, the sample shrinks to 1,925 apps, and this is used for the empirical analysis in Section 5.

3 Empirical Analyses

3.1 Descriptive Statistics of the IAP Subcategories

The finer classification of the IAP strategy into seven subcategories allows examination of trends in adoption of different IAP strategies. Table 1 shows summary statistics by entrant cohort (grouped by app release year, from 2010 until 2014) of the app performance measure and adoption of different IAP strategies.

The use of "freemium to remove features" shows a declining trend until 2012 but goes back up in 2013 and 2014 since many mobile apps started to have their IAA (in-app ads) removable. There are also upward trends in its combined uses of "freemium to add features"

[5] Within freemium, an example of addable features would be access to extra content, such as another level or a story in a game, shown in the lower screenshot of Figure A.2. An example of freemium to remove features would be one-time payment for disabling ads as in the third screenshot of Figure A.1.

[6] The first screenshot of Figure A.1 shows an example of subscription to add features: a monthly subscription that gives access to all contents and customized features in a workout app.

[7] The second screenshot of Figure A.1 shows an example of in-app credit: a phone call app in which users need to buy credits for calling and texting.

[8] An example of bundled IAP would be access to all the themes (shown in the upper screenshot of Figure A.2), also available to be sold separately (shown in the lower screenshot of Figure A.2).

and "bundled IAP" (from 7.0% and 1.1% in 2010 to 17% and 10% in 2014). This drove up adoption of more than one strategy (from 20% of the sample apps in 2010 to 40% in 2014). Another relevant upward trend is the use of "bundled IAP" (from 4.4% in 2010 to 19% in 2014).

The novel IAP strategy, "in-app currency," shows a steep increasing trend (from 21% in 2010 to 49% in 2014). However, there is a downward trend in the use of "freemium to add features," one of the earliest IAP strategies. The subscription and in-app credit strategies also show decreasing trends.

3.2 Peer Effects in Monetization Innovation Adoption

This section examines peer effects in adoption of the novel in-app currency (IAC) strategy. Trending adoption of the IAC strategy has been observed in the market, and how well this novel strategy works can be learned to some extent from how early adopters did relative to non-adopters.

I use cross-sectional data of mobile apps, pooled across app entrant cohorts (grouped by market entry year and quarters).[9] The dependent variable I_i^{IAC} is an indicator variable equal to one if app i adopts IAC upon its market entry. c_i denotes the cohort firm i belongs to. As a measure that quantifies the extent to which adoption is spread within some reference group of peers, I use the fraction of apps that adopt IAC in a given app category j of the prior cohort $c_i - 1$. The use of the prior cohort as a reference group is to avoid confounding by unobserved category-specific contemporaneous shocks that might affect adoption decisions across apps within an app category. The peer group adoption measure of app i of cohort c_i in app category j is:

$$\widetilde{I}_{j,c_i-1}^{IAC} = \frac{1}{n(S_{j,c_i-1})} \sum_{k \in S_{j,c_i-1}} I_k^{IAC}$$

where S_{j,c_i-1} is the set of apps in app category j in the prior cohort $c_i - 1$, and $n(S_{j,c_i-1})$ is the number of apps in S_{j,c_i-1}.

I estimate the probability that an app adopts IAC given the fraction of adopters among the peer apps, $\widetilde{I}_{j,c_i-1}^{IAC}$. The probability of IAC adoption by app i of cohort c_i in app category j is specified as follows:

$$Pr(I_i^{IAC}) = \beta \widetilde{I}_{j,c_i-1}^{IAC} + \gamma_j + \gamma_{c_i} + \varepsilon_i$$

where γ_{c_i} is cohort fixed effects, γ_j is app category fixed effects, and ε_i is the error term. I include app category fixed effects to account for the adoption costs that may vary across app categories due to intrinsic characteristics. Variation across apps within an app category comes from variation in market entry timing, i.e. different cohorts and peers. I also include cohort fixed effects to control for a market-wide adoption trend.

The empirical implication of peer effects is the response of an app's adoption decision to the adoption tendency of its peers, so I test whether apps are more likely to adopt IAC when more peer apps adopt, hence $\beta > 0$. On the other hand, the implication of negative peer

[9] Division of entrant cohorts is arbitrary, and grouping by 6 months shows robust results.

effects is a switching adoption pattern, i.e. more likely adoption followed or preceded by less adoption by peer apps and vice versa, hence $\beta < 0$. Finding either positive or negative peer effects does not narrow down to a single implication. A few non-mutually-exclusive implications of positive peer effects include learning across cohorts and growing consumer interests in the IAC apps.

Table 2 reports the estimate $\hat{\beta}$ with inclusion of different sets of controls. In Column 4, app rating is binned and controlled non-parametrically with fixed effects. Column 4 shows that in a given app category, 10% more adopters among the earlier entrant are associated with a 2.8% increase in the likelihood of an app's IAC adoption, suggesting intra-category peer effects. This finding is robust across different sets of controls. Using five other IAP strategies (freemium to add features, freemium to remove features, subscription to add features, subscription to remove features, and in-app credit), I perform placebo tests and report the results in Table A.1. For each strategy, I replicate the regression in Column 4 of Table 2 and find no evidence of peer effects. I find the suggestive evidence of positive peer effects only in adoption of the novel in-app currency strategy.

3.3 Monetization Innovation Adoption and App Ratings

Motivated by upward trends in adoption of both the novel in-app currency strategy and app ratings across app entrant cohorts in Figure 1, this section examines whether an app with a high app rating is more or less likely to adopt IAC.

Figure 2 characterizes the association of app's rating with the likelihood of adopting IAC and two other strategies, freemium to add features and freemium to remove features. Each binned scatter plot presents a relation between the adoption probability of a given strategy and app rating (after controlling for time fixed effects), and the slope of the fitted line is an OLS estimate when the adoption probability is regressed on app ratings. I find a positive correlation between app's IAC adoption and its rating: a 1-star rating increase is associated with 13% higher chances of IAC adoption. In contrast, I find negative correlations between adoption and app rating for the other two freemium strategies. Since apps would adopt monetization strategies to maximize profits, their adoption behavior and its association with app ratings implies some association of adoption profits with app ratings across monetization strategies, which is discussed more in the next section where I find the positive association between app's gain from IAC adoption and app rating. However, the results do not necessarily imply that higher ratings lower profits from the freemium strategies. Higher ratings may enhance profits from the IAC strategy relatively more than those from the other strategies.

3.4 Monetization Innovation Adoption and Performance

This section explores performance gains associated with adoption of the novel in-app currency strategy. Would apps of certain characteristics gain more from the in-app currency adoption than others? Would early entrants gain more or less than later entrants? To examine performance gains from the IAC strategy, I compare the performance of IAC adopters relative to non-adopters across 1) entrant cohorts and 2) app categories.

3.4.1 Performance across Entrant Cohorts

Motivated by an upward trend in adoption of the IAC strategy in Figure 1, I explore whether early cohort adopters do better or worse than later cohort adopters relative to non-adopters of respective cohorts. First, I find the average performance of IAC adopters is better than non-adopters. Across entrant cohorts, I compare the average performance of IAC adopters relative to non-adopters. As a performance measure, I use whether an app's revenues are greater than $5,000. Panel (b) of Figure 3 shows the fraction of apps that have generated more than $5,000 among adopters and non-adopters respectively, and I find the performance difference between adopters and non-adopters to be greater in the 2013 and 2014 cohorts than in the earlier cohorts.

To examine drivers of the greater performance gains from in-app currency adoption in late cohorts, I first explore changes in the composition of app categories and find the increasing presence of game apps; games were 49.8% of the 2010 cohort and 77.2% of the 2014 cohort in Table A.2. Limiting the sample to game apps, I find performance gains from adoption in games to be greater relative to other categories in Figure A.3, implying that the increasing presence of games partially explains the greater performance gains in the late cohorts.

As another potential driver, I explore app quality. Using app rating as a measure of app quality and an indicator that equals one if an app's revenues are greater than $5,000 as the dependent variable, I examine the association of performance gains from IAC adoption with app ratings. I empirically test whether differences in performance gains from IAC adoption across entrant cohorts can be explained by differences in app ratings across cohorts. Controlling for app ratings can reveal the extent to which app rating explains the performance gain differences across cohorts (Cook, Diamond, Hall, List, and Oyer, 2018; Gelbach, 2016; Palmer, 2015). Specifically, I estimate the following specification:

$$Pr(revenue_i > \$5K) = \sum_{t=2011}^{2014} \beta_t \cdot I_i^c \times I_{it} + \boldsymbol{X_i}\boldsymbol{\alpha} + \varepsilon_i$$

where i indexes apps, and t indexes entrant cohorts, i.e. market entry year; I_i^c is an indicator variable that equals one if app i adopts in-app currency; I_{it} is an indicator variable that equals one if app i is of entrant cohort t; $\boldsymbol{X}$ is a set of control variables, which include I_i^c, I_{it}'s, app category fixed effects, and app quality controls; and ε_{it} is the error term. Apps that enter the market in 2010 are the base cohort.

The coefficients of interest are β_t's that quantify the average performance differences between IAC adopters and non-adopters across cohorts. For the 2011-2014 cohorts, the coefficients β_t's capture how much better IAC adopters perform than non-adopters in a given cohort relative to the base cohort. Cohort fixed effects capture the average performance of a given cohort relative to the base cohort, and the IAC adoption dummy I_i^c captures the average performance of adopters relative to non-adopters in the base cohort.

In Table 4, Columns 1 and 3 estimate the given specification without the app quality controls to quantify the average performance gains from IAC adoption across entrant cohorts. The later cohorts show greater performance gains than the early cohorts conditional on app categories but not app quality. Conditioning on the app quality controls and re-estimating reveals the extent to which time-varying distributions of app quality explain better performance by later cohort adopters and the innovation adoption trends across cohorts. Columns

2 and 4 include the app rating controls and show the differences in performance gains across cohorts to decrease in magnitude and become insignificant, suggesting that app quality is a driver of the inter-cohort heterogeneity in adoption of the IAC strategy. However, the results do not rule out other possible drivers that may also be correlated with app rating and performance gains, such as consumers' growing interests in using the IAP apps over time.

A positive coefficient estimate for the interaction between app rating and the IAC adoption dummy suggests that higher quality is associated with more performance gains. Given profit-maximizing adoption decisions by apps, this finding is consistent with more likely IAC adoption by higher quality apps, discussed in Section 3.3.

3.4.2 Performance across App Categories

Adoption of the novel in-app currency strategy in the game category versus the music or productivity category may have different impacts on app performance depending on how well the strategy fits in with contents and features. Some monetization strategies may be inherently more or less attractive to some app categories than others. Table 3 shows that adoption of the IAC strategy has been more common in some categories, such as games, social networking, sports, and entertainment, whereas the strategy has not been adopted in some other categories, such as medical, weather, and finance.

As a measure of performance gains from IAC adoption, the last column of Table 3 reports the difference in the proportion of apps that have generated more than $5,000 revenues between IAC adopters and non-adopters in a given category. Visualized in Panel (a) of Figure 3, this relative performance measure is comparison of the average performance between the adopters and non-adopters. IAC adopters show particularly better performance than non-adopters in some categories, i.e. games, social networking, lifestyle, productivity, and utilities, relative to the other categories. For example, dating apps in the social networking category have adopted IAC and done well. In the productivity and utilities categories where the IAC strategy is not so popular, better performance of the adopters is due to a particular type of apps that sell Instagram "likes" and "followers" to users, categorization of which is questionable. Apart from such exceptions, adopters do better than non-adopters in categories where the IAC strategy is more popular, suggesting differences in suitability of the IAC strategy across categories.

Database is not big enough to have power to further examine which factor drives such performance differences across categories, but at the aggregate-level, data suggest that app quality is not a driver. For example, health & fitness and photo & video categories show very high ratings but not-so-good performance by IAC adopters.

4 Model of Monetization Strategy Adoption

In order to gain more economic insights, I develop a simple model. Given the short-lived nature of mobile apps, I build a simple two-period discrete choice model between novel and old monetization strategies. The model also reflects the empirical findings and some institutional features of the mobile app market to some extent. I present two scenarios of monetization strategy adoption choices: 1) a general case of novel and old monetization strategies and 2) a particular case of IAP and IAA. Discussions on modeling decisions and model implications follow.

4.1 Setting

I use a model of heterogeneity in app quality. For simplicity, each app developer is assumed to own a single app of quality $\theta \in [1, 5]$. To avoid confusion, I refer the decision maker as an "app" throughout the section. First, an app chooses an advertising investment for user acquisition based on an industry practice of advertising and buying downloads (Bresnahan, Li, and Yin, 2016).

4.1.1 User Acquisition

An app of quality θ decides advertising investments for periods 1 and 2, $a_1, a_2 \in [0, 1]$, to maximize the total monetizing profits, and the cost of the advertising investment $c(a_t)$ is strictly increasing and concave: $c'(a_t) > 0$ and $c''(a_t) \geq 0$. For simplicity, the marginal cost of advertising is assumed to be constant: $c'(a_t) = c$, $c > 0$.

The number of incoming users n_t in period t is decided by a function of an advertising investment a_t and market congestion β_t, i.e. how congested the mobile app market is: $n_t = f(a_t) = \beta_t a_t$. $\beta_t \in [0, 1]$ (higher β indicates lower congestion). A user base N_t is the summation between the user base and the incoming users of the previous period: $N_t = N_{t-1} + n_{t-1}$.

The app chooses $a_1, a_2 \in [0, 1]$ to maximize the total profits:

$$\Pi(\theta, N_0) = max_{a_1,a_2} \pi(\theta, N_1) - c(a_1) + \delta \{\pi(\theta, N_2) - c(a_2)\}$$

where $\delta \in [0, 1]$ is a discount rate, $N_0 \in [1, \infty]$, and $n_0 = 0$. The functional form of the periodic profit is $\pi(\theta, N_t) = \theta \log(N_t)$ where a user base enters through a log function to reflect the skewed distribution in usage and spending by users, i.e. concentrated on the upper tail. The dependence of the profit on app quality is to reflect the empirical findings from Section 3.

4.1.2 General Case of Monetization Innovation Adoption

The above base model is combined with an adoption choice between novel (N) and old (O) monetization strategies. The periodic profit function is:

$$\pi(\theta, N_t) \quad = \quad \alpha_i \theta \log(N_t) - c_i, i \in \{N, O\}$$

where parameters α_i and c_i denote the profit coefficient and adoption fixed costs of strategy i respectively ($\alpha_N > \alpha_O$, $c_N > c_O$: greater adoption fixed costs and the greater profit coefficient of the novel strategy). The adoption choice between the novel and old strategies is made in period 0 and irreversible.

4.1.3 Particular Case of Monetization Innovation Adoption: IAP and IAA

This particular setup sheds light on substitutability and complementarity between monetization strategies. Combined with an adoption choice between a novel and old strategies, IAP (P) and IAA (A) respectively, the periodic profit function is:

$$\begin{aligned}\pi(\theta, N_t) &= [\alpha_P(\theta - \lambda I\{A=1\})\log(N_t) - c_P]\, I\{P=1\} \\ &\quad + [\alpha_A \theta \log(N_t) - c_A]\, I\{A=1\}\end{aligned}$$

where coefficients α_A and α_P are the profit coefficients of IAA and IAP respectively, and c_A and c_P are the adoption fixed costs of IAA and IAP respectively ($\alpha_P > \alpha_A$, $c_P > c_A$). The former and latter terms are profits from IAP and IAA respectively, but the former includes the quality penalty for adopting both IAA and IAP: $\lambda \in [1, \frac{5\alpha_A}{\alpha_P})$; the app can adopt both but suffer from some app quality penalty since the app with ads would, ceteris paribus, be less preferred by users than one without. Other than the penalty, the setup is analogous to the general case.

In both the general and particular cases, the key in this model of app quality heterogeneity is the novel strategy's trade-off between higher fixed costs and greater performance with respect to app quality from the novel strategy. The novel strategy is not strictly preferred by apps of all θ's due to fixed costs.

4.2 Predictions

This section presents detailed discussions of the model implications.

4.2.1 User Acquisition

The app of quality θ has a user base of N_0, and faces the app market congestion β. For simplicity, Propositions 2-5 abstract from the corner solution case, the proofs of which is trivial.[10]

Lemma 1 *The app chooses* $a_1^* = \max\left\{\frac{\delta\theta}{c} - \frac{N_0}{\beta_1}, 0\right\}$ *and* $a_2^* = 0$, *and the total profits are:*

$$\Pi^*(\theta, N_0) = \max\left\{\theta\log(N_0) - \delta\theta + \frac{cN_0}{\beta_1} + \delta\theta\log(\frac{\beta_1\delta\theta}{c}), (1+\delta)\theta\log(N_0)\right\}$$

Proof. *In Appendix B.1* ■

An app (of quality θ) chooses the optimal advertising investment for user acquisition and monetizes the user base. The model makes the app choose the optimal investment a_1^* so that the period 2 user base is optimized to $N_2^* = N_1^* + n_1^* = \frac{\beta_1\delta\theta}{c}$ (which would be less for harsher discounting, more congested market, or more expensive advertising). The optimal period 2 user base N_2^* $(= \frac{\beta_1\delta\theta}{c})$ does not depend on the existing user base N_0. So, the incoming users are N_2^* less the existing user base: $n_1^* = \frac{\beta_1\delta\theta}{c} - N_0$. The exception would be the corner solution ($a_1^* = 0, n_1^* = 0$) where for the given quality and the market congestion, the app

[10] The corner solution would make the following wording changes in the propositions: from "increasing (decreasing)" to "non-decreasing (non-increasing)."

already has enough or more users than what the app would have wanted as its user base by period 2 ($N_0 > \frac{\delta\beta_1\theta}{c}$).

Of the closed form solution for the optimal total profits, the first term is the period 1 profit, the second and third terms are the optimal advertising investment costs, and the fourth term is the discounted period 2 profit. I derive empirically testable comparative statics below.

Proposition 2 *Given $\frac{\delta\theta}{c} - \frac{N_0}{\beta_1} > 0$, incoming users increase in quality but decrease in congestion and an existing user base.*
Proof. *In Appendix B.2* ■

The app makes the optimal investment a_1^* (of Lemma 1) at which the incoming users are the optimal period 2 user base less the existing user base: $n_1^* = N_2^* - N_0 = \frac{\beta_1\delta\theta}{c} - N_0$. Each existing user would lessen an additional incoming users ($\frac{\partial n_1^*}{\partial N_0} = -1$) so that the period 2 user base totals $\frac{\beta_1\delta\theta}{c}$. Without the existing user base N_0, the model predictions are less realistic, i.e. the app always needs to make some investment ($a_1^* > 0$).[11]

Since the optimal period 2 user base is the sum of the incoming users and the existing user base ($N_2^* = n_1^* + N_0$), the optimal period 2 user base N_2^* would increase in app quality and decrease in congestion ($\frac{\partial N_2^*}{\partial\theta} = \frac{\delta\beta_1}{c}$ and $\frac{\partial N_2^*}{\partial\beta_1} = \frac{\delta\theta}{c}$), and so would the incoming users ($\frac{\partial N_2^*}{\partial\beta_1} = \frac{\partial n_1^*}{\partial\beta_1}$ and $\frac{\partial N_2^*}{\partial\theta} = \frac{\partial n_1^*}{\partial\theta}$).

Proposition 3 *Given $\frac{\delta\theta}{c} - \frac{N_0}{\beta_1} > 0$, the total profits increase in quality and an existing user base and decrease in congestion. The marginal profit with respect to app quality is greater when there is lower congestion.*
Proof. *In Appendix B.3* ■

App quality increases the total profits through two channels, the period 1 profit and the optimal user acquisition, which adds onto the period 2 user base. The user base N_0 also affects the total profits through two channels, the period 1 profits and the costs reduction in user acquisition (i.e. the first and second terms respectively in $\frac{\partial\Pi^*}{\partial N_0} = \frac{\theta}{N_0} + \frac{c}{\beta_1}$). Saving the costs of acquiring the user base N_0 that it already has, the app only needs to acquire the remainder $N_2^* - N_0$.[12] Given the negative effect of congestion on incoming users in Proposition 2, congestion affects the total profits by making user acquisition harder.

The latter part of Proposition 3 stems from the interaction between app quality and a user base, i.e. better quality apps can monetize better across users to some degree, and implies that lower congestion in the app market would amplify the effect of app quality on the profits. The market condition can widen or narrow the performance gap between high- and low-quality apps, which is to some degree consistent with the early market where there was less congestion, and there were fewer apps on the "stickier" top charts.

[11] In a model without N_0, there would also be no period 1 profit. The problem simplifies to $\Pi(\theta) = max_{a_1} - ca_1 + \delta\theta\log(\beta_1 a_1)$, optimized at $a_1^* = \frac{\delta\theta}{c}, n_1^* = \frac{\beta_1\delta\theta}{c}$. So, $a_1^* > 0$

[12] Even in the corner solution where the app only monetizes the existing users without user acquisition, the app quality and the user base still increase the profits.

4.2.2 Monetization Innovation Adoption

Proposition 4 *Under some regularity conditions, there exists a quality threshold θ^* such that $\Pi_N(\theta^*) = \Pi_O(\theta^*)$. The app would adopt the novel strategy if $\theta \geq \theta^*$.*
Proof. *In Appendix B.4* ■

The app would adopt the novel strategy over the old strategy if $\Pi_N(\theta) > \Pi_O(\theta)$. Given the novel strategy's trade-off between greater profits with respect to app quality and higher fixed costs ($\alpha_N > \alpha_O$ and $c_N > c_O$), the implication of the model would be a threshold quality below which the app would prefer the old strategy over the novel strategy.

Proposition 5 *The marginal profit with respect to app quality from IAP adoption is greater than that from IAA adoption, and there exists a quality threshold above which the marginal profit with respect to app quality from adoption of both IAP and IAA is greater than that from adoption of either.*
Proof. *In Appendix B.5* ■

Consistent with the empirical findings in Section 3.4, i.e. better performance by adopters of higher quality, the profit from adoption of the novel IAP strategy is greater for higher quality. The dependence of app's adoption of monetization strategies on its quality is also consistent with the findings in Section 3.3. The model suggests a possibility of selective adoption of the monetization strategy innovation only by high-quality apps. The latter part of Proposition 5 suggests the association of the substitutability/complementarity of the strategies with app quality; despite the app quality penalty from adopting both strategies, higher quality apps may still adopt both.

5 Empirical Analyses of the Model

This section examines the empirical implications and consistency of the model.

I test the model predictions regarding incoming users and profits. I use non-monetizing mobile apps (e.g., corporate apps, banking apps, shopping apps, etc.) as a market congestion measure, app's rating as a quality measure, and the inverse of downloads and revenue rankings as measures for incoming users and profits respectively. Consistent with proposition 2, Table A.3 shows that incoming users are associated with app quality positively but with congestion negatively. Proposition 3 predicts app profits to increase in quality and decrease in congestion, and consistent correlations are reported in Table A.3.

5.1 Monetization Strategy Choices

This section estimates the model of adoption choice between novel and old monetization strategies, i.e. IAP and IAA respectively, through a multinomial logistic model of four discrete choices: IAA (A), IAP (P), both (B), or none(N) (McFadden, 1981). The variable I_i^j is an indicator variable that equals one if app i adopts strategy j. The app i's likelihood of adopting strategy j is specified as:

$$\Pr(I_i^j = 1|\theta_i) = \frac{\exp(\beta_\theta^j \theta_i + \beta_0^j)}{\sum_{k\in\{A,P,B,N\}} \exp(\beta_\theta^k \theta_i + \beta_0^k)}$$

where θ_i is app i's quality, measured by app's rating. Each app's log likelihood contribution is $l_i = \sum_{j\in\{A,P,B,N\}} I_i^j \ln \Pr(I_i^j = 1|\theta_i)$, and the overall log likelihood function is the sum of l_i across N apps, $lnL(\beta) = \sum_{i=1}^{N} l_i$. The parameters β's are estimated by maximizing $lnL(\beta)$. The base case is no monetization strategy ($j = N$).

A measurement error from the use of app rating as a measure of app quality would attenuate the coefficients. Consumer preference shocks, i.e. consumers' interests in apps with IAP and IAA, would affect both app ratings and app's adoption of those strategies. This would confound the coefficient estimates. Any other unobservables associated with app's rating and adoption behavior are also identification concerns.

Table 4 reports the estimation results. As an empirical analog with an interpretable scale, I compute the numerical derivatives of the choice probabilities with respect to app quality, $\frac{\partial \Pr(I_i^j=1|\theta_i)}{\partial\theta} |_{\theta=\overline{\theta}}$, at the mean app quality ($\overline{\theta} = 3.9$). A 1-star rating increase in app quality is associated with increases in the probability of adoption of both strategies by 17% and IAP by 0.59% but with decreases in the adoption probability of IAA by 4.5%. To some extent, the results are analogous to the findings in Section 3.3 that app quality is associated with adoption of the in-app currency strategy positively but with adoption of the other strategies negatively. Consistent with Proposition 5, Figure 4 shows positive numerical derivatives of the adoption probability of both strategies, suggesting some complementarity between IAA and IAP (e.g. IAAs removable through IAP).

5.2 Dynamics in Monetization Innovation Adoption

This section is to introduce some dynamics to the static equilibrium setup. I assume that app quality θ has a log-logistic distribution with parameter γ and c.d.f. F.[13] When θ^* is implicitly defined such that $\Pi_N(\theta^*) = \Pi_O(\theta^*)$, the proportion of apps that adopt the novel strategy would be $1 - F(\theta^*)$. The threshold quality in entrant cohort t is specified as:

$$\theta^*(t) = \theta_0^* e^{-\lambda t}$$

where λ is the exponential rate at which the threshold declines. Given the log-logistic distribution $F_t(\theta)$ for cohort t,

$$\begin{aligned} 1 - F_t(\theta^*) &= e^{-\gamma ln\theta^*(t)} F_t(\theta^*) \\ ln\{\frac{1 - F_t(\theta^*)}{F_t(\theta^*)}\} &= (\gamma\lambda)t - \gamma ln\theta_0^* \end{aligned}$$

The above equation is estimated simply by a regression of the empirical log odds ratio from data on time period (by 6 months). I recover $\hat{\lambda}$ from the intercept and the slope estimates: $\hat{\lambda} = .13$.[14] The estimate suggests that the adoption quality threshold decreases by 13% from the prior period. At the aggregate level, I suggest dynamics through a declining adoption quality threshold and an accompanied increasing trend in adoption, as in Figure

[13] Griliches (1957) first used this specification for the diffusion of innovation adoption.

[14] The intercept and slope are estimated using OLS: $\hat{\gamma\lambda}$=.16*** (.023), $\hat{\lambda} \ln \theta_0^*$= 1.2*** (.12). The slope parameter would be a multiplication between the rate λ at which the threshold falls and the shape parameter γ of the distribution, and I set the threshold among the first entrants as $\ln \theta_0^* = 1$.

1 though I cannot identify how exactly app's adoption decision changes over time since different micro-level changes can be observationally equivalent at the aggregate level.

6 Conclusion

This paper examines monetization strategy innovations in the mobile app market. I examine the adoption and impacts of the innovations to better understand what types of apps choose which monetization strategies. In empirical analyses, I exploit adoption of in-app currency, which is a relatively novel strategy and shows an increasing trend in adoption.

I find suggestive evidence of positive peer effects within app category in adoption of IAC but not of other conventional monetization strategies, suggesting a few non-mutually-exclusive implications including information imperfection, learning across cohorts, and growing consumer interests in the IAC apps. By comparing adopters and non-adopters, I also find the positive association of app ratings with adoption tendency and profits. Moreover, better performance by later entrant adopters relative to the early ones is to some degree driven by better app ratings of the late entrants though I cannot rule out other drivers that may be correlated with app rating and performance, such as consumers' growing interests in using the IAP apps over time. Adoption of the IAC strategy has been more common in some categories than others, and I find that adopters do better than non-adopters in categories where the strategy is more popular.

In order to gain more economic insights, I develop a simple model of app quality heterogeneity and suggest a possibility that a novel monetization strategy could selectively benefit and be adopted by high-quality apps, supported by estimation of a logistic adoption choice model. I also suggest some dynamics in the adoption process.

Throughout the paper, the results suggest that monetization strategy innovations in the mobile app market may not benefit all types of apps, which may be the case in other innovations so could be taken into consideration in designing policy programs to promote innovations.

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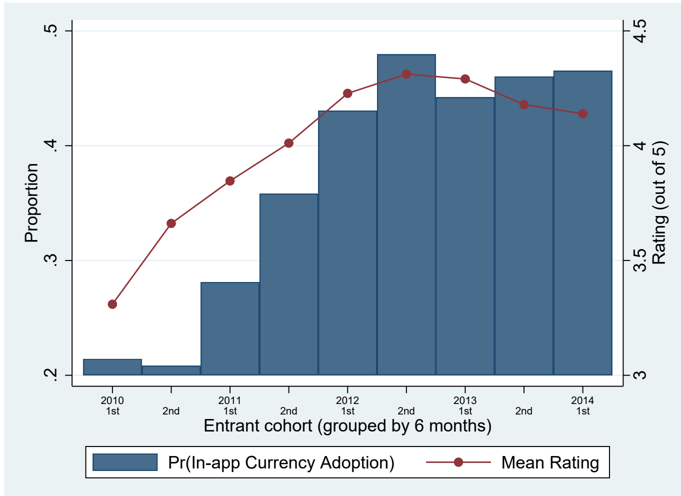
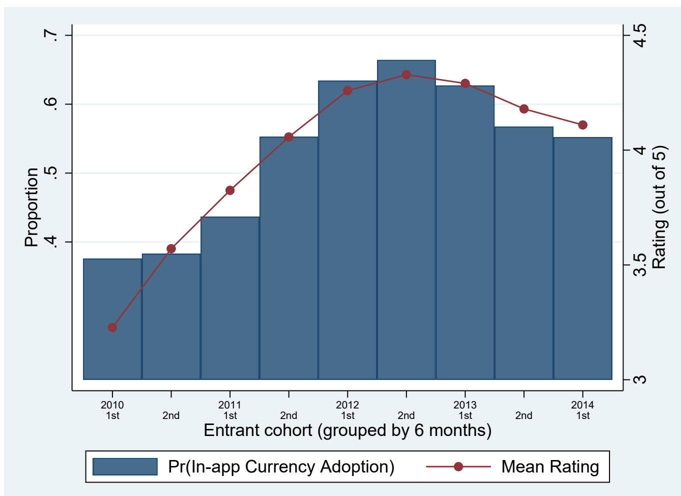
Figure 1: In-app Currency Adoption and App Rating by Entrant Cohort



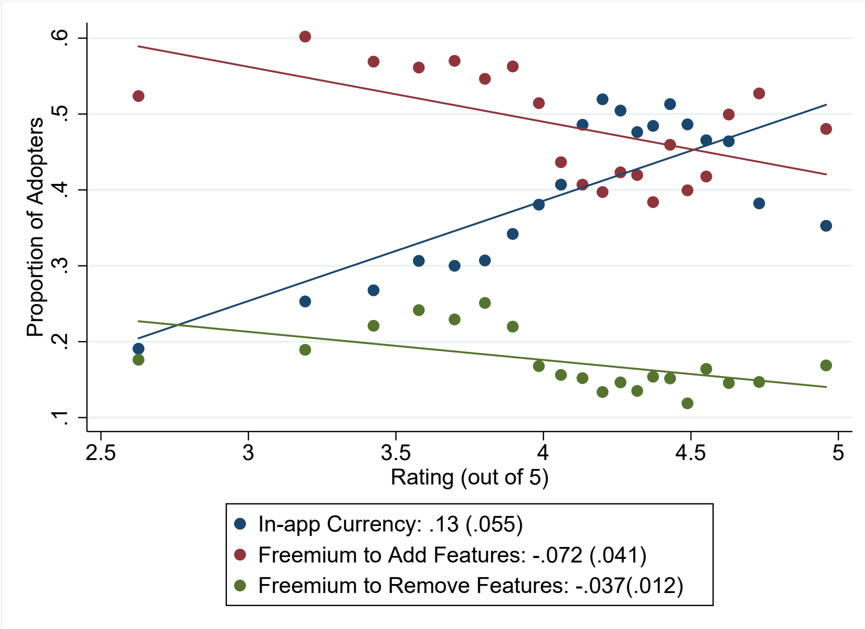
(a) All Categories



(b) Games

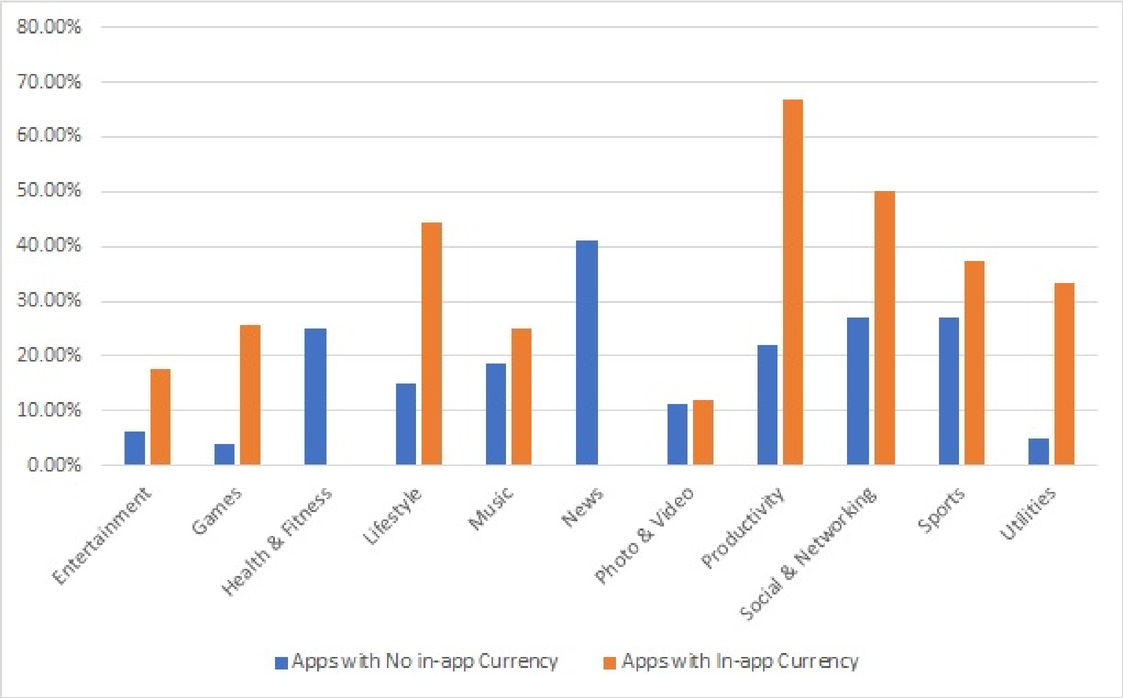
These figures use a 6-month time window. In a given period, each bar is the proportion of entrants that adopt in-app currency, and each point is the mean of average ratings: across all categories in (a) and for games in (b).

Figure 2: Monetization Strategy Adoption and App Quality

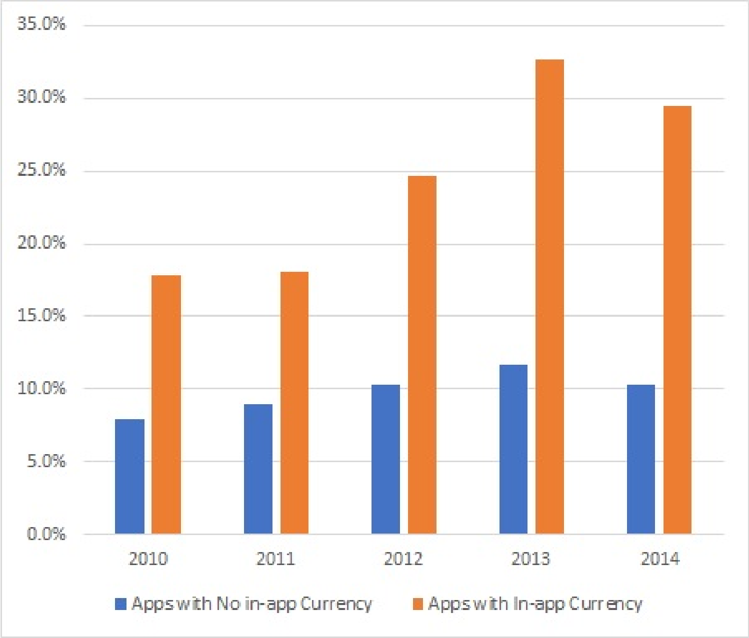


This figure shows binned scatter plots of the relationship between the likelihood of a given strategy adoption and the app rating across mobile apps. In each panel, app's rating is binned into 20 quantiles and plotted on the x-axis. The proportion of apps that adopt the given strategy is plotted on the y-axis. I control for time fixed effects. I estimate the fit lines on the binned points using OLS and report the slope coefficients and standard errors (in parentheses).

Figure 3: In-app Currency Adoption and Performance



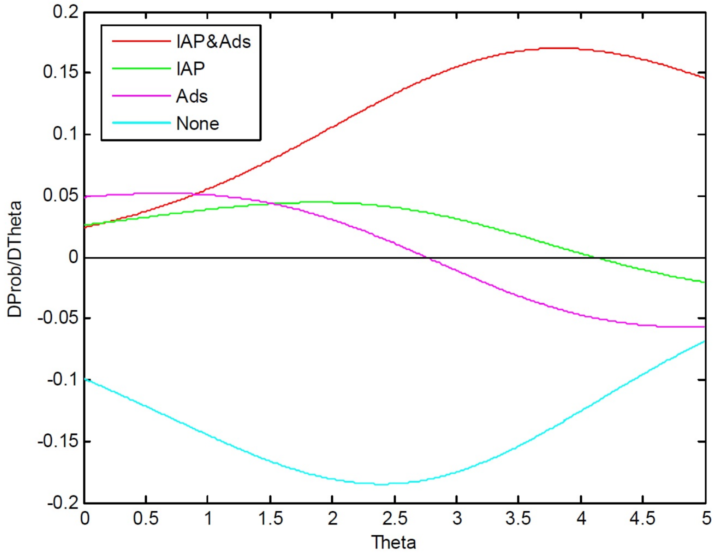
(a) Categories



(b) Entrant Cohorts

These figures show the proportion of apps that have generated more than $5,000 revenues among in-app currency adopters (an orange bar) and non-adopters (a blue bar) respectively: across categories in (a) and cohorts in (b).

Figure 4: Numerical derivative of Monetization Strategy Adoption Probabilities



This figure visualizes the estimation results of the multinomial logit model, reported in Table 5, with an interpretable scale. I compute the numerical derivatives of the adoption probabilities with respect to app quality at the mean app quality ($\overline{\theta} = 3.9$).

Table 1: Summary Statistics of Monetization Strategies by App Cohort

	All years 6,158 apps	2010 828 apps	2011 1,522 apps	2012 1,579 apps	2013 1,403 apps	2014 826 apps
Performance						
I{Revenue > $5,000}	16.1%	10.0%	12.0%	16.9%	21.2%	19.6%
Monetization Strategies						
Freemium to add features (AF)	48.5%	55.8%	52.2%	45.5%	45.8%	44.7%
Freemium to remove features (RF)	17.3%	16.7%	15.4%	14.4%	17.2%	27.5%
Subscription to add features (AF)	5.1%	6.2%	6.6%	5.7%	3.8%	2.7%
Subscription to remove features (RF)	0.4%	1.4%	0.4%	0.4%	0.1%	0.0%
In-app credit	1.1%	1.3%	1.1%	1.2%	0.9%	0.7%
In-app currency	39.4%	21.0%	32.8%	45.7%	45.1%	48.5%
Bundled IAP	9.6%	4.4%	5.1%	7.0%	15.3%	18.8%
Freemium AF × Freemium RF	10.4%	7.0%	8.9%	8.9%	12.3%	16.6%
Freemium RF × Bundled IAP	3.8%	1.1%	1.5%	2.2%	5.9%	10.4%
Currency × Freemium AF	8.8%	4.2%	7.6%	10.5%	9.8%	10.4%
Currency × Freemium RF	4.6%	2.1%	4.0%	5.5%	4.8%	6.3%
Credit × Subscription AF	0.2%	0.4%	0.0%	0.3%	0.2%	0.0%
Credit × Subscription RF	0.1%	0.2%	0.0%	0.2%	0.1%	0.0%
Multiple strategies	31.3%	20.3%	27.5%	29.6%	38.5%	39.9%

This table shows summary statistics by app cohort of a performance measure, an indicator of whether an app has generated cumulative revenue greater than $5,000 and a set of indicators for an app's adoption of monetization strategies.

Table 2: Peer Effects in In-app Currency Adoption

DEP VAL: Pr(In-app Currency Adoption)	(1)	(2)	(3)	(4)
Fraction of In-app Currency Adopters	.957***	.576***	.378***	.278***
Among Peers	(.0140)	(.151)	(.0900)	(.0616)
App Category FE	No	Yes	Yes	Yes
Time FE	No	No	Yes	Yes
App rating Control	No	No	No	Yes
Number of Apps	6,091	6,091	6,091	6,091
R^2	.248	.255	.261	.298

*** p<0.01, ** p<0.05, * p<0.1

This table shows the results of a regression of app's in-app currency adoption decision on the fraction of adopters among peer apps, i.e. apps in the same app category in the prior entrant cohort. Time is in quarters (3 months). App rating is binned and non-parametrically controlled with fixed effects in Column 4. Standard errors in parentheses are clustered by app category.

Table 3: In-app Currency Adoption across Categories

Category	Total number of apps	In-app currency adoption (%)	Relative performance (%)
Catalogs	9	11.1	0
Education	162	3.1	-17.9
Entertainment	337	13.4	11.6
Games	4,054	56.6	21.7
Health & Fitness	118	1.7	-25.0
Lifestyle	122	7.4	29.4
Music	153	5.2	6.4
News	40	2.5	-41.0
Photo & Video	354	4.8	0.5
Productivity	171	1.8	44.6
Social Networking	145	20.7	23.0
Sports	56	14.3	10.4
Utilities	150	4.0	28.5

This table shows the proportion of apps that adopt in-app currency across app categories. Relative performance is defined as the difference in the proportion of apps that have generated more than $5,000 revenues between in-app currency adopters and non-adopters in a given category. Omitted categories where there is no app that adopts in-app currency are books, business, finance, food & drink, medical, navigation, reference, shopping, Travel, and Weather.

Table 4: Response of Performance to In-app Currency Adoption across Entrant Cohorts

DEP VAL: Pr(Revenues > $5K)	(1)	(2)	(3)	(4)
2011 Cohort	-.00822	-.0483	-.0107	-.0433*
× In-app Currency = 1	(.0351)	(.0331)	(.0274)	(.0248)
2012 Cohort	.0452	-.0228	.0461*	-.0272
× In-app Currency = 1	(.0340)	(.0314)	(.0268)	(.0236)
2013 Cohort	.111**	.0362	.0974**	.0181
× In-app Currency = 1	(.0435)	(.0397)	(.0379)	(.0338)
2014 Cohort	.0921*	.0249	.0641	-.00478
× In-app Currency = 1	(.0450)	(.0446)	(.0427)	(.0416)
Rating		.0497***		.0397***
		(.0128)		(.0112)
Rating		.0893***		.0970***
× In-app Currency = 1		(.0147)		(.0159)
Time FE	Yes	Yes	Yes	Yes
App Category FE	No	No	Yes	Yes
Number of Apps	6,158	6,158	6,158	6,158
R^2	.0547	.0703	.0871	.101

*** p<0.01, ** p<0.05, * p<0.1

This table shows the results of a regression of app's performance measure, i.e. an indicator of whether an app has generated more than $5,000 revenues, on the in-app currency adoption dummy interacted with the entrant cohort dummies and app rating. App categories are controlled in Columns 3 and 4. Standard errors in parentheses are clustered by app category.

Table 5: Estimation of Monetization Strategy Choice Model

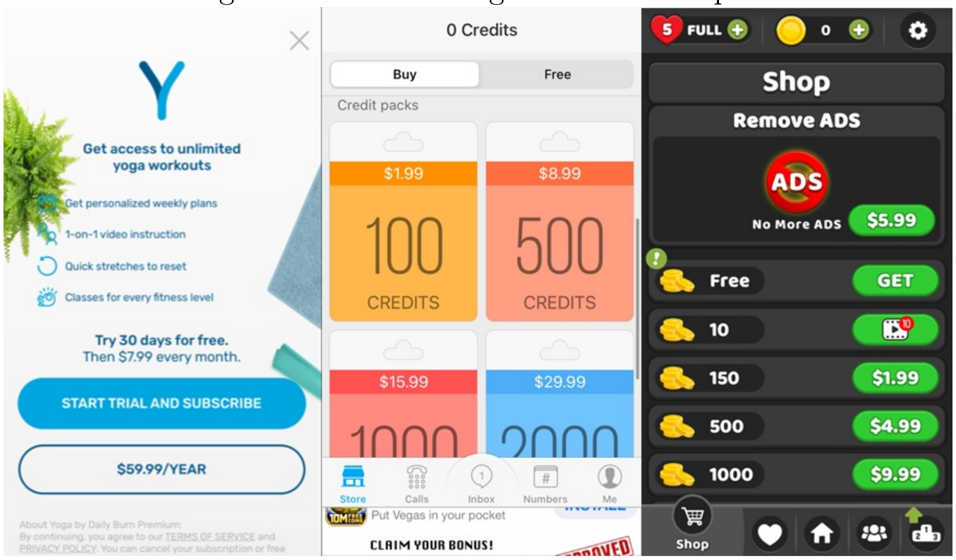
Strategy Adoption Choices	Parameters	
IAP and IAA	App Quality	1.10*** (.0990)
	Constant	-3.48*** (.383)
IAP	App Quality	.731*** (.116)
	Constant	-2.93*** (.448)
IAA	App Quality	.487*** (.104)
	Constant	-1.78*** (.396)
Number of observations	1,925	

*** p<0.01, ** p<0.05, * p<0.1

This table reports the parameter estimates of the multinomial logit model of monetization strategy adoption choices among IAP, IAA, both, and none. The base case is adoption of none. Standard errors are in parentheses.

A Appendix: Figures and Tables

Figure A.1: IAP Categorization Examples

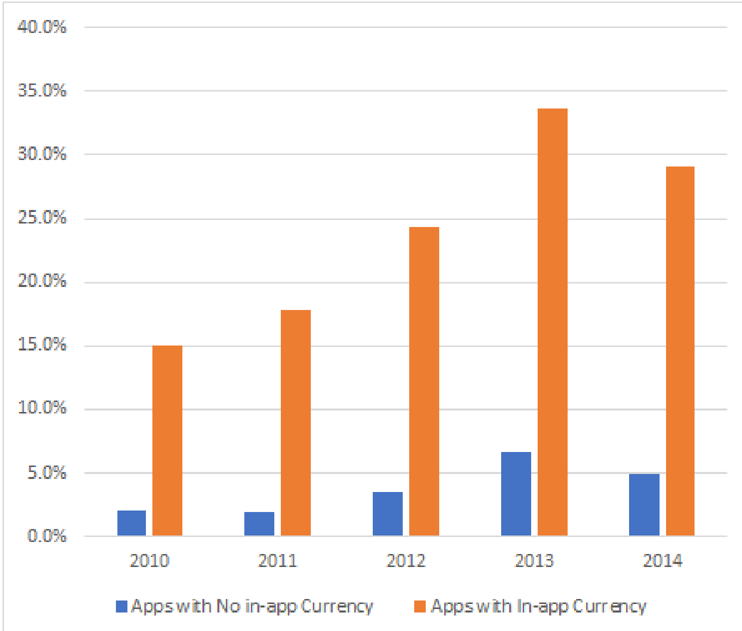


This figure shows examples of mobile app's in-app purchase options to users. Based on the categorization scheme of the MIG database, three screenshots from left to right are examples of: subscription to add features (from "Yoga Workouts by Daily Burn"), in-app credits (from "Text Me - Phone Call + Texting"), and freemium to remove features and in-app currency (from "Gordon Ramsay: Chef Blast").

Figure A.2: Example of Bundled In-app Purchases

This figure shows an example of a mobile app's in-app purchase options to users: freemium to add features and bundled in-app purchases (from "Tizi World - My Pretend Life").

Figure A.3: In-app Currency Adoption and Performance in Games



This figure shows the proportion of game apps that have generated more than $5,000 revenues among in-app currency adopters (an orange bar) and non-adopters (a blue bar) respectively across app entrant cohorts.

Table A.1: Placebo Test Results: Peer Effects

DEP VAL: Pr(Adoption) Strategy:	(1) Freemium AF	(2) Freemium RF	(3) Subscription AF	(4) Subscription RF	(5) Credit
Fraction of Adopters	.0154	.0517	.0549	.210	.104
Among Peers	(.0584)	(.0639)	(.0824)	(.129)	(.0752)
App Category FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
App rating Control	Yes	Yes	Yes	Yes	Yes
Number of Apps	6,091	6,091	6,091	6,091	6,091
R^2	.0985	.0348	.194	.0390	.0825

*** p<0.01, ** p<0.05, * p<0.1

This table shows the placebo test results of the peer effects in adoption of the in-app currency strategy (Column 4 of Table 2). The adoption probabilities of five other monetization strategies are regressed on the fraction of adopters among peer apps, i.e. apps in the same app category in the prior entrant cohort. Time is in quarters (3 months). App rating is binned and non-parametrically controlled with fixed effects. Standard errors in parentheses are clustered by app category.

Table A.2: App Category Composition by Entrant Cohort

Cohort (by year)	2010	2011	2012	2013	2014
Books	1.3	0.9	1.0	0.4	0.4
Business	1.1	0.7	0.4	0.6	0.2
Catalogs	0.0	0.1	0.4	0.1	0.0
Education	3.6	3.0	3.1	1.9	1.3
Entertainment	9.1	6.3	4.8	4.1	4.0
Finance	0.6	0.3	0.3	0.4	0.1
Food & Drink	0.6	0.3	0.4	0.3	0.1
Games	49.8	61.8	66.8	71.9	77.2
Health & Fitness	2.8	1.6	2.2	2.3	0.6
Lifestyle	3.1	3.4	1.4	1.1	0.9
Medical	1.0	0.4	0.2	0.2	0.0
Music	4.1	3.3	2.1	1.8	1.3
Navigation	0.9	0.6	0.6	0.2	0.1
News	1.3	0.8	0.6	0.2	0.5
Photo & Video	4.5	6.4	5.6	6.3	5.2
Productivity	4.5	2.6	3.1	1.9	2.2
Reference	1.1	0.6	0.8	0.5	0.4
Shopping	0.1	0.0	0.1	0.1	0.0
Social & Networking	4.0	2.2	2.2	1.9	2.1
Sports	1.2	0.8	1.1	0.8	0.6
Travel	1.5	0.7	0.4	0.4	0.2
Utilities	3.1	2.7	2.2	2.0	2.4
Weather	0.9	0.5	0.3	0.5	0.1
Number of apps	828	1,522	1,579	1,403	826

This table breaks down each entrant cohort by app category. Each entry is the proportion of apps in a given category such that each column adds up to 100.

Table A.3: Correlations with App Quality and Congestion Measures

DEP VAL:	Incoming Users (1)	Profits (2)
App Quality	.00106***	.0124***
	(.0000887)	(.000163)
Congestion	-.000110***	.000107***
	(.000000513)	(.00000260)
Number of Observations	558,980	558,980
R^2	.0805	.0165

*** p<0.01, ** p<0.05, * p<0.1

Column 1 shows the association of incoming users with app quality and congestion, and Column 2 shows the association of profits with app quality and congestion. Each observation is the app by day level.

B Appendix: Proofs

B.1 Proof of Lemma 1

Proof. In period 2, the app developer will not invest because $c'(a_t) > 0$, and choosing $a_2 > 0$ has zero effect on revenues: $a_2^* = 0$.
The periodic profit π_2 is increasing and concave in a_1.

$$\frac{\partial \pi_2}{\partial a_1} = \frac{\partial \pi_2}{\partial N_2}\frac{\partial N_2}{\partial a_1} = \frac{\theta}{N_2}\beta_1 > 0$$
$$\frac{\partial^2 \pi_2}{\partial a_1^2} = \frac{\partial^2 \pi_2}{\partial N_2^2}\left(\frac{\partial N_2}{\partial a_1}\right)^2 = -\frac{\theta}{\partial N_2^2}\beta_1^2 < 0$$

The advertising cost is linear, so $c''(a_t) = 0$, and the objective function is also concave in a_1.
From the first order condition with respect to a_1,

$$-c'(a_1) + \delta\frac{\partial \pi_2}{\partial a_1} = 0$$
$$-c + \delta\frac{\beta_1\theta}{N_0 + \beta_1 a_1} = 0$$

By solving the FOC,

$$a_1^* = \frac{\delta\theta}{c} - \frac{N_0}{\beta_1}$$

By plugging in a_1^*, the solution becomes:

$$\begin{cases} a_1^* = \max\left\{\frac{\delta\theta}{c} - \frac{N_0}{\beta_1}, 0\right\} \\ a_2^* = 0 \\ \Pi^* = \max\left\{\theta\log(N_0) - \delta\theta + \frac{cN_0}{\beta_1} + \delta\theta\log(\frac{\beta_1\delta\theta}{c}), (1+\delta)\theta\log(N_0)\right\} \end{cases}$$

■

B.2 Proof of Proposition 2

Proof. Given $\frac{\delta\theta}{c} - \frac{N_0}{\beta_1} > 0$, the interior solution will be chosen for the optimal advertisement investment: $a_1^* = \frac{\delta\theta}{c} - \frac{N_0}{\beta_1}$.
By differentiating a_1^* with respect to θ, β_1, and N_0,

$$\frac{\partial a_1^*}{\partial \theta} = \frac{\delta}{c} > 0$$
$$\frac{\partial a_1^*}{\partial \beta_1} = \frac{N_0}{\beta_1^2} > 0$$
$$\frac{\partial a_1^*}{\partial N_0} = -\frac{1}{\beta_1} < 0$$

$\frac{\partial n_1^*}{\partial \theta} = \beta_1\frac{\partial a_1^*}{\partial \theta} > 0$, $\frac{\partial n_1^*}{\partial \beta_1} = a_1^* + \beta_1\frac{\partial a_1^*}{\partial \beta_1} > 0$ (lower β_1 is higher congestion), and $\frac{\partial n_1^*}{\partial N_0} = \beta_1\frac{\partial a_1^*}{\partial N_0} < 0$ ■

B.3 Proof of Proposition 3

Proof. Given $\frac{\delta\theta}{c} - \frac{N_0}{\beta_1} > 0$, the interior solution will be chosen: $\Pi^* = \theta\log(N_0) - \delta\theta + \frac{cN_0}{\beta_1} + \delta\theta\log(\frac{\beta_1\delta\theta}{c})$.
By differentiating Π^* with respect to θ, β_1, and N_0,

$$\begin{aligned}
\frac{\partial\Pi^*}{\partial\theta} &= \log(N_0) + \delta\log(\frac{\beta_1\delta\theta}{c}) > \log(N_0) + \delta\log(N_0) \geq 0 \\
\frac{\partial\Pi^*}{\partial\beta_1} &= \frac{c}{\beta_1}(\frac{\delta\theta}{c} - \frac{N_0}{\beta_1}) > 0 \\
\frac{\partial\Pi^*}{\partial N_0} &= \frac{\theta}{N_0} + \frac{c}{\beta_1} > 0
\end{aligned}$$

So, $\frac{\partial\Pi^*}{\partial\theta} > 0$, $\frac{\partial\Pi^*}{\partial N_0} > 0$, and $\frac{\partial\Pi^*}{\partial\beta_1} > 0$ (the lower β_1, the higher congestion).

For the second part of the proposition, I take the cross derivative of the profits with respect to quality and congestion:

$$\frac{\partial\Pi^*}{\partial\theta\partial\beta_1} = \frac{\delta}{\beta_1} > 0$$

Since lower β_1 represents higher congestion, $\frac{\partial\Pi^*}{\partial\theta\partial\beta_1} > 0$ proves the second part. ■

B.4 Proof of Proposition 4

Proof.

The strategy-dependent profit coefficients in the periodic profit and adoption fixed costs are the only changes, so the second order condition still holds. The proofs assuming the interior solutions are below:

Case 1 ($O = 1$): The profit is $\pi_t = \alpha_O\theta\log(N_t) - c_O$, and by solving the first order condition of π with respect to a_1,

$$\begin{aligned}
a^*_{1,A} &= \frac{\delta\alpha_O\theta}{c} - \frac{N_0}{\beta_1}(\geq 0 \text{ for the interior solution}) \\
\Pi^*_O &= \alpha_O\theta\log(N_0) - \delta\alpha_O\theta + \frac{cN_0}{\beta_1} + \delta\alpha_O\theta\log(\frac{\beta_1\delta\alpha_O\theta}{c}) - c_O(1+\delta)
\end{aligned}$$

Case 2 ($N = 1$): The profit is $\pi_t = \alpha_N\theta\log(N_t) - c_N$, and by solving the first order condition of π with respect to a_1,

$$\begin{aligned}
a^*_{1,P} &= \frac{\delta\alpha_N\theta}{c} - \frac{N_0}{\beta_1}(\geq 0 \text{ for the interior solution}) \\
\Pi^*_N &= \alpha_N\theta\log(N_0) - \delta\alpha_N\theta + \frac{cN_0}{\beta_1} + \delta\alpha_N\theta\log(\frac{\beta_1\delta\alpha_N\theta}{c}) - c_N(1+\delta)
\end{aligned}$$

Define $F(\theta) = \Pi^*_N - \Pi^*_O$, and take its derivative

$$
\begin{aligned}
\frac{\partial F}{\partial \theta} &= \frac{\partial \Pi_N^*}{\partial \theta} - \frac{\partial \Pi_O^*}{\partial \theta} \\
&= \log(N_0)(\alpha_N - \alpha_O) + \delta\{\alpha_N \log(\frac{\beta_1 \delta \alpha_N \theta}{c}) - \alpha_O \log(\frac{\beta_1 \delta \alpha_O \theta}{c})\} \\
&> 0
\end{aligned}
$$

Under the following regularity conditions $\frac{(c_N - c_O)(1+\delta)}{(\alpha_N - \alpha_O)(log(N_0)-\delta)} > \theta_{max}$ and $\frac{c_N - c_O}{\alpha_N(1+log(\frac{\alpha_N \theta max}{c}))} > \theta_{min}$, the following can be shown:

$$
\begin{aligned}
F(\theta_{min}) &= \Pi_N^* - \Pi_O^* \mid_{\theta=\theta_{min}} < 0 \\
F(\theta_{max}) &= \Pi_N^* - \Pi_O^* \mid_{\theta=\theta_{max}} > 0
\end{aligned}
$$

Since $\frac{\partial \Pi_N^*}{\partial \theta} - \frac{\partial \Pi_O^*}{\partial \theta} > 0$, there exist $\theta^* \in [\theta_{min}, \theta_{max}]$ such that $\Pi_N^* - \Pi_O^* \mid_{\theta=\theta^*} = 0$
■

B.5 Proof of Proposition 5

Proof. The penalty coefficient and the strategy-dependent profit coefficients in the periodic profit are the only changes, so the second order condition still holds. The proofs of the interior solutions are below (the proofs of the corner solutions are trivial so omitted):

Case 1 ($A = 1$ and $P = 0$): The profit is $\pi_t = \alpha_A \theta \log(N_t) - c_A$, and by solving the first order condition of π with respect to a_1,

$$
\begin{aligned}
a_{1,A}^* &= \frac{\delta \alpha_A \theta}{c} - \frac{N_0}{\beta_1} (\geq 0 \text{ for the interior solution}) \\
\Pi_A^* &= \alpha_A \theta \log(N_0) - \delta \alpha_A \theta + \frac{c N_0}{\beta_1} + \delta \alpha_A \theta \log(\frac{\beta_1 \delta \alpha_A \theta}{c}) - c_A(1+\delta)
\end{aligned}
$$

Case 2 ($A = 0$ and $P = 1$): The profit is $\pi_t = \alpha_P \theta \log(N_t) - c_P$, and by solving the first order condition of π with respect to a_1,

$$
\begin{aligned}
a_{1,P}^* &= \frac{\delta \alpha_P \theta}{c} - \frac{N_0}{\beta_1} (\geq 0 \text{ for the interior solution}) \\
\Pi_P^* &= \alpha_P \theta \log(N_0) - \delta \alpha_P \theta + \frac{c N_0}{\beta_1} + \delta \alpha_P \theta \log(\frac{\beta_1 \delta \alpha_P \theta}{c}) - c_P(1+\delta)
\end{aligned}
$$

Case 3 ($A = 1$ and $P = 1$): The profit is $\pi_t = \{\alpha_P(\theta - \lambda) + \alpha_A \theta\} \log(N_t) - (c_P + c_A)$, and by solving the first order condition of π with respect to a_1,

$$
\begin{aligned}
a_{1,A\&P}^* &= \frac{\delta\{\alpha_P(\theta - \lambda) + \alpha_A \theta\}}{c} - \frac{N_0}{\beta_1} (\geq 0 \text{ for the interior solution}) \\
\Pi_{A\&P}^* &= (\log(N_0) - \delta)\{\alpha_P(\theta - \lambda) + \alpha_A \theta\} + \frac{c N_0}{\beta_1} \\
&\quad + \delta\{\alpha_P(\theta - \lambda) + \alpha_A \theta\} \log(\frac{\beta_1 \delta\{\alpha_P(\theta - \lambda) + \alpha_A \theta\}}{c}) \\
&\quad - (c_P + c_A)(1+\delta)
\end{aligned}
$$

If I take the derivatives of the profits with respect to θ respectively,

$$\begin{aligned}
\frac{\partial \Pi_A^*}{\partial \theta} &= \alpha_A \log(N_0) + \delta\alpha_A \log(\frac{\beta_1 \delta \alpha_A \theta}{c}) \geq \alpha_A \log(N_0) + \delta\alpha_A \log(N_0) \geq 0 \\
\frac{\partial \Pi_P^*}{\partial \theta} &= \alpha_P \log(N_0) + \delta\alpha_P \log(\frac{\beta_1 \delta \alpha_P \theta}{c}) \geq \alpha_P \log(N_0) + \delta\alpha_P \log(N_0) \geq 0 \\
\frac{\partial \Pi_{A\&P}^*}{\partial \theta} &= (\alpha_A + \alpha_P) \log(N_0) + \delta(\alpha_A + \alpha_P) \log(\frac{\beta_1 \delta \{\alpha_P(\theta - \lambda) + \alpha_A \theta\}}{c}) \\
&\geq (\alpha_A + \alpha_P) \log(N_0) + \delta(\alpha_A + \alpha_P) \log(N_0) \\
&\geq 0
\end{aligned}$$

Now, I need $\frac{\partial \Pi_P^*}{\partial \theta} - \frac{\partial \Pi_A^*}{\partial \theta} > 0, \forall \theta$, and $\exists \overline{\theta}$ $\frac{\partial \Pi_{A\&P}^*}{\partial \theta} - \frac{\partial \Pi_P^*}{\partial \theta} > 0, \forall \theta > \overline{\theta}$.

$$\begin{aligned}
& \frac{\partial \Pi_P^*}{\partial \theta} - \frac{\partial \Pi_A^*}{\partial \theta} \\
&= \log(N_0)(\alpha_P - \alpha_A) + \delta\{\alpha_P \log(\frac{\beta_1 \delta \alpha_P \theta}{c}) - \alpha_A \log(\frac{\beta_1 \delta \alpha_A \theta}{c})\} \\
&> 0
\end{aligned}$$

$\frac{\partial \Pi_P^*}{\partial \theta} - \frac{\partial \Pi_A^*}{\partial \theta} > 0, \forall \theta$ because $\beta_P > \alpha_A$.
Define $F(\theta)$ as follows:

$$\begin{aligned}
F(\theta) &= \frac{\partial \Pi_{A\&P}^*}{\partial \theta} - \frac{\partial \Pi_P^*}{\partial \theta} \\
&= \log(N_0)\alpha_A + \delta(\alpha_A + \alpha_P) \log(\frac{\beta_1 \delta \{\alpha_P(\theta - \lambda) + \alpha_A \theta\}}{c}) \\
&\quad - \delta\alpha_P \log(\frac{\beta_1 \delta \alpha_P \theta}{c})
\end{aligned}$$

$$\begin{aligned}
\frac{\partial F(\theta)}{\partial \theta} &= \frac{\delta(\alpha_A + \alpha_P)^2}{\{\alpha_P(\theta - \lambda) + \alpha_A \theta\}} - \frac{\delta \alpha_P}{\theta} \\
&> \frac{\delta(\alpha_A + \alpha_P)^2}{(\alpha_A + \alpha_P)\theta} - \frac{\delta \alpha_P}{\theta} \\
&= \frac{\delta \alpha_A}{\theta} > 0
\end{aligned}$$

So, $\frac{\partial F(\theta)}{\partial \theta} > 0$.
Since $\lambda < \frac{5\alpha_A}{\alpha_P}$,

$$\begin{aligned}
\alpha_P(5 - \lambda) + 5\alpha_A &> 5\alpha_P \\
\log(\frac{\beta_1 \delta \{\alpha_P(5 - \lambda) + 5\alpha_A\}}{c}) &> \log(\frac{5\beta_1 \delta \alpha_P}{c}) \\
\delta(\alpha_A + \alpha_P) \log(\frac{\beta_1 \delta \{\alpha_P(5 - \lambda) + 5\alpha_A\}}{c}) &> \delta\alpha_P \log(\frac{5\beta_1 \delta \alpha_P}{c})
\end{aligned}$$

Using the above inequality and $\log(N_0)\alpha_A > 0$,

$$\begin{aligned} F(5) &= \log(N_0)\alpha_A + \delta(\alpha_A + \alpha_P)\log(\frac{\beta_1\delta\{\alpha_P(5-\lambda) + 5\alpha_A\}}{c}) \\ &\quad -\delta\alpha_P \log(\frac{5\beta_1\delta\alpha_P}{c}) \\ &> 0 \end{aligned}$$

Since $F(5) > 0$ and $\frac{\partial F(\theta)}{\partial\theta} > 0$, there exists $\overline{\theta} \in [1, 5]$ such that $F(\theta) = \frac{\partial\Pi^*_{Ads\&P}}{\partial\theta} - \frac{\partial\Pi^*_P}{\partial\theta} > 0, \forall\theta > \overline{\theta}$. ■